Friedrich-Ebert-Gymnasium, Bonn

Gymnasium der Stadt Bonn mit internationalem Profil

Bilingual deutsch-französischer Bildungsgang und bilingual deutsch-englische Bildungswege

School code: 000973

Mathematics Analysis and Approaches SL & HL including the German Mathematics Curriculum completed in two years

The mathematics curriculum of the IB Diploma Programme and the national curriculum are combined and taught simultaneously at our school. Though candidates benefit from common requirements and synergetic effects, several topics are only covered by one curriculum. Candidates take three to five 45-minute-classes per week depending on the level they choose in the German course of education and two 45-minute-classes to prepare them for the additional requirements of the IB Diploma Programme.

Teachers who completed this outline:

Melanie Blum / Gabriele Josten

Date of last IB training: 21/22 September 2019

Resources:

Mathematics: Analysis and Approaches SL. OUP (2019)
Mathematics: Analysis and Approaches HL. OUP (2019)
Lambacher Schweizer Mathematik Qualifikationsphase, Grund- und Leistungskurs
Old exam questions
Mathematics: analysis and approaches HL formula booklet (first assessment 2024)
Mathematics: analysis and approaches SL formula booklet (first assessment 2024)
Das Große Tafelwerk - Formelsammlung, Cornelsen (2003)
Geogebra
Excel
Graphic Display Calculator TI Nspire CX

Course outline

The national curriculum is expanded on the additional topics as indicated in the second column of the table. While the SL students work on exercises on the topics, additional thematic content is dealt with with the HL students.

	National Syllabus Content Standard Level and Advanced Level	**Additional Analysis and Approaches Content Standard Level and Higher Level**
Year 1: Semester 1	**Algebra and Calculus** **Properties of Functions with parameters** Revision: domain, range, notation, graphs of functions, linear and quadratic functions, solving quadratic equations, transformations, differentiation of polynomial functions, limits and convergence, the derivative function, tangents, normals *TOK:* *Is mathematics just the manipulation of symbols under a set of rules? How can causal relationships be established in mathematics?* Differentiation rules (Graphical) interpretation of first, second and third derivatives, concavity, points of inflection Applications of differential calculus: optimization and kinematics Functions with parameters, Modelling with polynomial functions *TOK:* *What value does the knowledge of limits have? What is the link between mathematical models and physical reality? Mathematics - invented or discovered?* Process-oriented competences: modelling, problem solving, reasoning and using mathematical tools **Integration** Antiderivatives and the indefinite integral Area and definite integrals Properties of definite integrals Fundamental theorem of calculus Area between two curves Applications of integration Process-oriented competences: reasoning communication and using mathematical tools *TOK:* *What value does the knowledge of limits have? An infinite area sweeps out a finite volume. How does this compare to intuition? Is it possible to know about infinity?*	**Sequences and Series** Number patterns and sigma notation Arithmetic sequences and series. Geometric sequences and series. Applications. TOK: *Is mathematics a (universal) formal language? Do the names that we give things impact how we understand them? To what extent does the language we use shape the way we think? Is all knowledge concerned with identification and use of pattern?* Binomial theorem Proofs *TOK* *What is the difference between inductive and deductive reasoning? How does mathematical proof differ from reasoning in everyday life? Is mathematics beautiful?* **(Additional Higher Level:** Counting principles, extension of the binomial theorem and further proofs) **Functions:** The reciprocal function Transforming the reciprocal function Rational functions Special functions and their graphs Classification of functions Operations with functions Inverse functions *TOK: Is zero the same as nothing? Why does zero not have a reciprocal?* **(Additional higher Level** Function transformation Further rational functions) **Trigonometry** Geometry of 3D shapes Volumes and surfaces Right-angled triangle trigonometry The sine and cosine rule Applications

	Exponential Function **(Advanced Level: first semester, Standard Level: second semester)** Exponents, logarithms Derivatives of exponential functions natural logarithmic function Modelling exponential functions (only advanced level) Process-oriented competences: modelling, problem solving, reasoning and using mathematical tools. *TOK:* *How can a mathematical model give us knowledge if it does not yield accurate predictions?*	 *TOK:* *How can mathematics, being after all a product of human thought which is independent of experience be appropriate to the objects of the real world? Does mathematics start in our brains or is it part of the universe? Does it give us genuine knowledge of the world?*
Year 1: Semester 2	**Operations with functions & composite functions** Inverse functions (only advanced level) Further differentiation rules Integration techniques Modelling with functions Process-oriented competences: modelling, problem solving, reasoning and using mathematical tools **Vector Algebra:** **Lines** Vectors, points and equations of lines Scalar product and properties Angles and distances Applications and modelling Process-oriented competences: modelling and using mathematical tools **Planes** **(Advanced Level: second semester, Standard Level: third semester)** Solving systems of linear equations Parametric equations of planes Angles, distances and intersections Applications and modelling (Advanced level only: Vector product and properties Normal vector Distances of points, planes and lines) *TOK* *If we can find solutions in higher dimensions can we reason that these spaces exist beyond our sense of perception?*	**Trigonometry and trigonometric functions** Radian measure, arcs, sectors and segments Trigonometric ratios in the unit circle Trigonometric identities and equations Trigonometric functions Derivatives with sine and cosine Integration with sine, cosine *TOK:* *To what extent is mathematics a product of human social interaction.* *Is music mathematical?* **(Additional Higher Level:** **Complex Numbers** Further trigonometric ratios, identities and equations Further derivatives of trigonometric functions Polynomial equations Complex numbers The fundamental theorem of algebra Operations with complex numbers in polar form Powers and roots of complex numbers in polar form)

	Process-oriented competences: modelling, problem solving and using mathematical tools	
Year 2: Semester 1	**Statistics and Probability:** Sampling Representation of data Measures of central tendency Measures of dispersion Theoretical and experimental probability Diagrams Sample spaces Conditional probability Dependent and independent events Discrete random variables Expected value, standard deviation and variance Binomial distribution (Advanced level only: Significance Testing Interval Estimates and Confidence Normal distribution) Process-oriented competences: modelling, problem solving reasoning, communication and using mathematical tools *TOK* *Is it fair to sell lottery tickets? How is intuition used in mathematics? How do mathematics reconcile the fact that some conclusions conflict with intuition?* *Do ethics play a role in mathematics?*	**Statistics for bivariate data and Probability** Scatter diagrams, measuring correlation Line of best fit Least squares regression Random variables The normal distribution *TOK:* *How easy is it to be misled by statistics? Is there a difference between information and data? How reliable are mathematical measures? To what extent can we rely on technology? Do we create mathematics to emulate real-life situations or is the world mathematical?* **More calculus:** Derivatives of the natural logarithmic function Applications of derivatives Differentiation and integration techniques Kinematics and accumulating change **Additional higher level:** Ordinary differential equations Implicit differentiation and related rates Inequalities Limits
Year 2: Semester 2	**Stochastic processes** Matrix representation Matrix multiplication Limits Application **Integrated and overarching revision and consolidation**	**Integrated and overarching revision and consolidation**

IB internal assessment requirement to be completed during the course

Students and their parents are rendered general information on internal assessments in all subjects within the IB Diploma Programme during the two years preceding the start of the first year. An overview of the topics covered and examples of former explorations are presented during an information session on all IB subjects three months before the start of the IB Diploma Programme. After the first quarter of the first year students are asked to mark an exploration from the support material. Each group marks the exploration according to one or two assessment criteria and prepares feedback and suggestions on how this particular piece of work could be improved. The assessment criteria, the reason for the marks awarded by the students and proposals for improvement are then presented and discussed in detail. All general hints, notes and advices are collected so that they can serve as guidance for the students´ own exploration. Ways to find appropriate topics and successful methods to handle these are discussed in class. Students are provided with access to further material on the exploration on MS Teams including examples of the work of our graduates, requirements and recommendations. During the second quarter of the first year students are asked to start searching for areas of mathematics they would like to focus on in their exploration. At the beginning of the third semester a further exploration from the support material is marked in class to ensure that students are reminded of the concept of the exploration and the assessment criteria. Students decide on a topic after the third quarter and submit their first draft before the summer break. After getting feedback, students have two weeks to submit their final version in the first quarter of the second year.

Links to TOK - An example of how links between the topics of mathematics and TOK are explored.

When introducing the topic integration, a wide range of applications are discussed both in order to reach a deeper understanding of the concepts involved and to arouse the students` curiosity and motivation. Students get the opportunity to discover the fundamental theorem of calculus themselves in class. It is then formulated as a conjecture and proven together. Various applications of the fundamental theorem such as determining areas and volumes as well as examples from physics are covered.
When calculating the work required to place a satellite in orbit using the integral of force with respect to displacement, then determining the work required to shoot this object infinitely far into space and considering the geometrical interpretation of this physical problem in a coordinate system the students are confronted with an astonishing phenomenon. The area between the graph of the function f(x)= $c \cdot \frac{1}{x^2}$ (c>o) and the x-axis from a (a>0) to infinity is finite while the lengths of the curve is infinite. Calculating the volume of Torricelli´s trumpet and estimating the area of its surface applying integration techniques one obtains a similar astonishing result: The object generated by rotating the infinitely large area about the x-axis has a finite volume. These calculations lead to several questions: What value does the knowledge of limits have? How do these results relate to intuition? Is it possible to know about infinity? When first discovered, Torricelli´s trumpet was considered a paradox and raised doubts about mathematical concepts and dictums. However, it did not only have an impact on mathematics but also on philosophy causing a dispute between mathematicians and philosophers on the concept of infinity. Students learn about the historical context and are confronted with the painter´s paradox. They experience mathematical knowledge as a result of a struggle for truth and the influence that discoveries in one discipline have on others.

Approaches to learning

An example from one topic of how students develop learning skills

To introduce the regression line, students are given a concrete example of the problem of making a prediction about the value of a certain quantity (e.g. the presumed size of the tallest or shortest person given that the shoe size is known). Students have the opportunity to find creative solutions independently and in groups. Data can be collected or researched and be visualised using technology. When developing a suitable model function, students use their knowledge of (linear) functions and the mean as a measure of central tendency. By comparing their own models with those of their classmates, naming the advantages and disadvantages and considering what conditions an optimal model function has and how the conditions can be expressed mathematically, students develop a deep understanding of the topic. Moreover, discussing their ideas in groups and presenting their optimal model they develop both social and communication skills. Students are encouraged to critically analyse the model functions and reflect upon their quality. Further tasks trigger reflection on when and in which cases the chosen approach is appropriate. The students' own considerations also motivate them to gain an understanding of the regression line produced by the graphic display calculator and the significance of the correlation coefficient, thus further research on this and for example covariance is encouraged.

Links to CAS

High-achieving students in mathematics who are in the IB Diploma Programme at our school offer a wide range of support services for lower-achieving students. For example, individual tutoring is offered for younger pupils. Moreover, as part of their CAS activities, students organised a weekly maths club for younger students who have difficulties in this subject. IB Diploma candidates also support teachers in mathematics in our international classes to help immigrant children.
Furthermore, IB students support mathematical competitions that are held at our school and help assessing the work of younger students. In addition, some students also get actively involved in promoting excellence in mathematics at our school.
Many of our IB Diploma candidates are also members of the student council, where, in addition to calculating the costs of events, the creation and evaluation of surveys plays an important role. This is also relevant in some of the clubs offered at our school. For example, a survey was conducted and statistically analysed by IB students who were members of the school´s Fairtrade Club. They had campaigned for the sale of Fairtrade products in the school's own kiosk and evaluated how these products were accepted by the student body.

International mindedness

An example of a topic which contributes to international mindedness

The universal language of mathematics can be seen as a common and therefore international language that is understood by people all over the world. Mathematics fosters and supports the further development and discovery of knowledge in a wide variety of fields and disciplines all over the world. Mathematical models in both the social sciences and the natural sciences help to investigate issues that are relevant to all people in society as a whole. The example of algebra and the concept of a function illustrates that today's knowledge and notations have been developed through the work of numerous mathematicians from a wide range of cultures. The ancient Babylonians and Egyptians already studied quadratics, as were Arabic and Hindu mathematicians. The foundations for the concept of a function were developed by Persian, Indian and Greek thinkers. The concrete development of the concept of a function also spans many countries, including France, Germany and Switzerland represented for example by Réne Descartes, Gottfried Wilhelm Leibzig and Leonard Euler.

Development of the IB learner profile

An example of how the contents and related skills of one topic pursue the development of the attributes.

Having investigated limits, average and instantaneous rates of change, the students make assumptions about derivatives of different kinds of functions as well as predictions about necessary differentiation rules (Inquirers). The method of differentiation from first principle allows them to prove simple rules like the constant rule, the constant multiple rule and the sum or difference rule themselves. When dealing with further differentiation rules, they make conjectures (Risk-takers) and develop their own methods to check their assumptions (Thinkers). Students investigate the graphical interpretation of the first and second derivative using technology. When working with partners they are open to their classmates' suggestions and discuss different approaches (Open-minded). Students reproduce and understand proofs they have researched and explain them step by step to their classmates (Communicators). They recognize the importance of differentiation in other disciplines such as Physics or Economics and transfer their knowledge to corresponding problems and applications (Knowledgeable). Especially when analysing the graph of exponential function modelling realistic phenomena like the spread of a disease or rising temperatures due to climate change students do not only critically evaluate the model but also discuss moral and ethical questions raised by the model (Principled). Students develop, compare, evaluate and reflect upon different approaches for optimization problems (Reflective). In doing so, they draw on knowledge from other areas of mathematics. They explain their approaches in presentations and partner or group work and support each other (Caring). When revising and preparing for exams students take breaks and engage e.g., in sports activities to balance out the workload (Balanced).

Promoting self-management and research skills:

Particularly when planning, researching and writing the exploration or an extended essay, the focus is on training self-management and research skills such as setting and adhering to deadlines, prioritising subtasks to be completed, refining time management, formulating precise questions, writing structured texts and using correct citation methods. Self-management skills in particular are continuously expanded and practised in preparation for the regular written examinations. In addition, independent work in class, working on complex problems and application tasks continuously challenge students to carry out smaller research projects, understand mathematical texts, and to apply problem-solving techniques such as searching for patterns, plan creation and critical questioning of methods and strategies.

Last reviewed: July 2024